UDC 532.526

OPTIMAL BOUNDARY LAYER SUCTION ON A POROUS PLATE WITH ALLOWANCE FOR INITIAL FLOW TURBULENCE

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Inzhenerno-Fizicheskii Zhurnal, Vol. 9, No. 2, pp. 151-154, 1965

It is shown that the initial flow turbulence has an appreciable influence on the optimal suction of fluid from the boundary layer on a porous flat plate.

The problem of optimal suction of fluid from the boundary layer on a porous flat plate was first solved in [1] by a numerical method. An analytical solution has been obtained by the author [2]. Other papers [3-5] have also been devoted to solving a similar problem for a boundary layer with a longitudinal pressure drop at its outer edge.

Analysis of experimental data [6] shows that a boundary layer may be laminarized by suction of fluid through the porous surface of an airfoil profile, if the local Re number equals its critical value at the transition point, which depends on the initial flow turbulence. In practice this becomes important in connection with calculations of the laminar boundary layer on airfoil profiles moving in media with small initial turbulence.

Attempts to evaluate the possible influence of initial turbulence on the optimal distribution of the rate of suction of small amounts of fluid through the specially prepared porous surfaces of airfoil profiles showed that such an evaluation is very awkward and laborious because of computational difficulties. It proved to be possible, however, to evaluate the principal part of the effect of initial turbulence on optimal suction from the boundary layer of airfoil profiles by solving the analogous problem for a porous plate.

A semi-empirical theory has been proposed [7] for the influence of initial turbulence on the transition from a laminar to a turbulent boundary layer. From the results of this paper the critical value of Re at the transition point for a plate may be written as follows:

$$\mathrm{Re}_t^{**} = \mathrm{Re}_l^{**} + c_1 \sqrt{f_s}/\varepsilon^{5/4}. \tag{1}$$

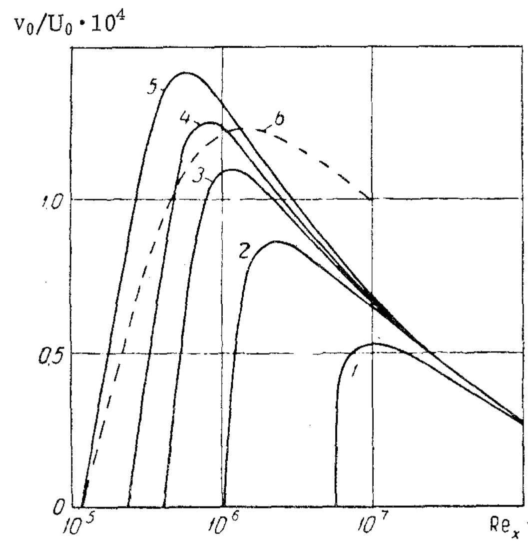
In determining the value of the lower critical Reynolds number Re_l^{**} it is recommended that the following approximate formula [2] be used:

$$\mathrm{Re}_l^{**} = \exp(A - BH). \tag{2}$$

We also transform the second term in (1) into an explicit function of the shape factor H. It is known [8] that the value of the shape factor at the separation point of a laminar boundary layer, f_s, and also the shape factor H, may be calculated using the following approximate relations:

$$f_s = f_{s_0} - kt^{**}, \tag{3}$$

$$H = H_0 + ct^{**}. \tag{4}$$



Dependence on Re number of distribution of optimal rate of suction of fluid from the boundary layer of a porous plate: 1—ε = 0.2; 2—0.5; 3—1; 4—2; 5—5; according to [1].

Substituting (2), (3), and (4) into (1) and making simple algebraic transformations, we finally obtain the local critical Re number at the transition point

$$\mathrm{Re}_t^{**} = \exp(A - BH) + \frac{c_1}{\varepsilon^{5/4}} \left[f_{s_0} - \frac{k}{c}(H - H_0) \right]^{1/2}. \tag{5}$$

Henceforth, in determining the dependence of the local Re number for the boundary layer on a porous plate on the shape factor H, the calculation will be carried out in a rectangular coordinate system. We locate the origin of the coordinate system at the leading edge of the plate, the x axis being along the plate surface, and the y axis normal to it. Applying the law of momentum variation to an element of the boundary layer on a porous plate in an incompressible fluid, we obtain the integral relation [2]

$$\mathrm{Re}^{**}\frac{d\,\mathrm{Re}^{**}}{d\,\mathrm{Re}_x}+t^{**}=\zeta. \tag{6}$$

Moreover, it follows from [8] that

$$\zeta=\zeta_0-dt^{**}. \tag{7}$$

Substituting (4) and (7) into (6) and making simple transformations, we obtain an ordinary differential equation for the local Re number for the boundary layer

$$\frac{d\,\mathrm{Re}^{**^2}}{d\,\mathrm{Re}_x}=2\zeta_0+\frac{2\,(d+1)\,H_0}{c}-\frac{2\,(d+1)}{c}H. \tag{8}$$

Taking H as parameter and satisfying the boundary condition $\mathrm{Re}^{**}=\mathrm{Re}_0^{**}$ for $\mathrm{Re}_x=\mathrm{Re}_{x_0}$, we may write the integral of (8) as

$$\mathrm{Re}^{**^2}=\left[2\left(\zeta_0+\frac{d+1}{c}H_0\right)-\frac{2\,(d+1)}{c}H\right](\mathrm{Re}_x-\mathrm{Re}_{x_0})+\mathrm{Re}_0^{**^2}. \tag{9}$$

It should be noted that Re_0^{**} and Re_{x0} in (9) are values of the local Re number at the transition point for a plate without boundary layer suction. The value Re_0^{**} for a given degree of initial flow turbulence, ε, is calculated from (5), taking $H = H_0$.

Further calculations were carried out as follows. The calculations began at the transition point. From (9) the relation $\mathrm{Re}^{**}=\mathrm{Re}^{**}(\mathrm{Re}_x, H)$ was calculated for given values of H, and then the corresponding critical Re number at the transition point, $\mathrm{Re}_t^{**}(\varepsilon, H)$, was calculated from the known initial turbulence according to (5). Next, eliminating this parameter, we obtained the function H for the case when the local Re number for the boundary layer is equal to its critical value at the transition point. Since the parameter t^{**} has a single-valued relation to H, [Eq. (4)], the distribution of the rate of suction of fluid from the boundary layer of the porous plate was calculated as follows:

$$v_0/U_0=t^{**}/\mathrm{Re}^{**}. \tag{10}$$

The optimal suction velocity distribution for the plate is shown in the figure as a function of the Re number and the initial turbulence. It follows from an analysis of the data presented that the optimal suction velocity distribution and the total suction rate depend appreciably on the initial turbulence. For example, the maximum dimensionless optimal suction velocity for initial flow turbulence $\varepsilon = 2\%$ is $v_0/U_0 = 1.24\times10^{-4}$, while for $\varepsilon = 0.2\%$ the corresponding value is 0.56×10^{-4}. Naturally, the lower the initial turbulence, the less fluid needs to be sucked from the boundary layer through the porous surface of the plate.

NOTATION

x and y—coordinates along and normal to the plate surface; U_0— free stream velocity; $v_0(x)$—distribution of normal velocity component over plate surface (local suction velocity); δ^{**} and δ^*—momentum thickness and displacement thickness of boundary layer; ν and ρ—kinematic viscosity and density of fluid; $\mathrm{Re}^{**}=U\delta^{**}/\nu$ and $\mathrm{Re}_x=U_x/\nu$—local Reynolds numbers; Re_l^{**} and $\mathrm{Re}_{x_0}^{**}$— values of lower critical Reynolds number; Re_t^{**}—critical Reynolds number at transition point; $t^{**}=v_0\delta^{**}/\nu$—parameter relating to suction of fluid from boundary layer; $H=\gamma^*/\delta^{**}$— boundary layer shape factor; τ_ω—local friction stress on plate surface; $\zeta=\tau_w\delta^{**}/\nu_p U_0$—dimensionless local friction coefficient; ε—degree of initial flow turbulence; $c_1 = 1.88$; $A = 31.3$; $B = 10$; $f_{s0} = 0.089$; $k = 0.194$; $H_0 = 2.59$; $c = 1.18$; $\zeta_0 = 0.22$; $d = 0.56$—constants.

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6 November 1964

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